

Chaotic self-sustaining structure embeded in turbulent-laminar interface

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(Dated: March 17, 2015)

An interface structure between turbulence and laminar flow is investigated in two-dimensional channel flow. This spatially localized structure not only sustains itself, but also converts laminar state into turbulence actively. In other words, this coherent structure has a functionality to generate inhomogeneity by its inner dynamics. The dynamics of this functional coherent structure is isolated using the filtered simulation, and a physical perspective of its dynamics is summarized in a phenomenological model called an “ejection-jet” cycle, which includes multiscale interaction process.

PACS numbers: 47.27.ed, 47.10.Fg, 47.27.nd, 05.45.Jn

INTRODUCTION

Turbulence ubiquitously appears in nature: from quark-gluon plasma [1] to the Universe [2]. Because of its strong nonlinearity, most studies related to turbulence may have adopted more or less statistical or coarse graining approaches [3]. Though they have vividly revealed phenomenological and/or kinematic natures of turbulence such as the energy transfer among different scales and places, these statistical treatments are not sufficiently adequate to elucidate concrete mechanisms of even such fundamental processes of turbulence: For example, what substance, e.g. vortices, transfers energy or why the energy transfer occurs. On the other hand, the dynamical systems approaches to turbulence have helped us describe these mechanisms with numerically obtained components (invariant sets) in the phase space such as fixed points, periodic orbits and their connections [4].

Recent developments in the dynamical systems approach to turbulence arrive at the next stage, where the spatial inhomogeneity is taken into account. Famous actors on the previous stage are the “minimal” flows [5], which mean direct numerical simulations with minimal system sizes reproducing elementary processes and some statistical quantities of turbulence. The phase spaces embedding them are effectively low-dimensional. However, those of the spatially inhomogeneous turbulent flows are no longer low-dimensional, and it is quite hard to treat such high-dimensional phase spaces both theoretically and numerically.

One simple strategy to overcome this high-dimensionality is to consider spatially localized self-sustaining structures as building blocks (BBs) of turbulence. Though each of BBs may consist of internal fundamental elements, each block is expected to be effectively low-dimensional like the minimal flows. Indeed, various types of numerical exact solutions to the Navier-Stokes equation corresponding to localized coherent structures have been obtained so far in pipe flow [6], plane Couette flow [7–9] and asymptotic suction boundary layer [10]. Then the dynamics of spatially extended systems are expected to be decomposed into

that of each localized coherent structure and their interactions.

At first glance this building block strategy may be incompatible with global inhomogeneity since they are introduced to represent local dynamics. One scenario to treat global inhomogeneity in this strategy is to regard it as a collective dynamics among BBs. Since each of BBs is represented by a low-dimensional model, a coarse-grained model governing their interactions can be deduced as done for the chemical oscillations [11]. This phase reduction [11] scenario has succeeded in explaining properties of “puff” in pipe flow [12]. We demonstrate, however, that this scenario breaks down at least for a fundamental inhomogeneous system, namely turbulent-laminar interface.

Instead, we try to deal with global inhomogeneity by extending roles of each BB. We focus on a turbulent-laminar interface in two-dimensional channel flow. As clarified in this paper, a localized self-sustaining structure is embedded in the interface. This structure, which we call chaotic interface (CI), produces turbulence downstream by its inner dynamics while invading upstream laminar flow. The turbulent-laminar interface is governed by CI in this sense; i.e. this global inhomogeneity is generated by the local dynamics. We introduce the term “functional” coherent structures (FCS) to represent such active localized coherent structures. We clarify the dynamics of CI in detail and evaluate how this functional building block scenario explains this global inhomogeneity.

CHAOTIC INTERFACE STRUCTURE

Two-dimensional laminar channel flow has the same critical Reynolds number Re_c as three-dimensional one. In 2D case, the TS-wave solution appearing at this critical point bifurcates into a weak chaotic state, which we call chaotic TS-wave, as its Reynolds number increases [13–16]. In this paper, we consider a channel which contains turbulent downstream region and laminar upstream region.

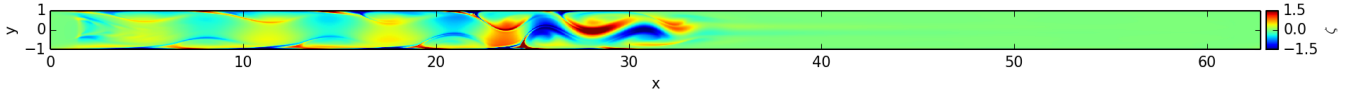


FIG. 1. A snapshot of turbulent vorticity field. ζ varies from -25 to 25 on the walls, and regions $\zeta > 1.5$ ($\zeta < -1.5$) are colored by the same color of $\zeta = 1.5$ ($\zeta = -1.5$).

We adopt a frame of reference moving at a speed c_I against the laboratory frame for CI not to march. The streamwise and the wall-normal coordinates are denoted by x and y , respectively in this interface frame. The system is non-dimensionalized by the half width of the channel, so $y \in [-1, 1]$. \mathbf{u} denotes the velocity field in this frame. We deal with a very long box $[0, 20\pi] \times [-1, 1]$ periodic in x , to emulate the dynamics realized in an infinitely long channel. Since the walls move in the interface frame, the non-slip boundary conditions become $\mathbf{u}(x, \pm 1) = -c_I \hat{\mathbf{x}}$, where $\hat{\mathbf{x}}$ denotes the x directional unit vector. The Reynolds number Re is fixed to 8000 in this paper.

To analyze the dynamics of this process in a finite computational box, we have to keep supplying laminar region since the turbulent region becomes wider as time goes on. We resolve this problem using the damping filter [17] in the interface frame. We introduce a linear damping term into the incompressible Navier-Stokes (NS) equation to reproduce a laminar Poiseuille flow $\mathbf{U}_L = (1 - y^2 - c_I) \hat{\mathbf{x}}$ in a small region $\Omega = [0, 1.4] \times [-1, 1]$:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} - H_{\sigma^2, \Omega}(x) (\mathbf{u} - \mathbf{U}_L),$$

$$H_{\sigma^2, \Omega}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\Omega} dx' \exp\left(-\frac{(x - x')^2}{2\sigma^2}\right),$$

where the last term of NS equation is the damping filter term. Since $c_I = 0.855$ is faster than the phase velocity of the chaotic TS-wave, this damping term laminarizes it, and the laminarized flow returns upstream due to the periodic boundary condition.

In this setting a turbulent-laminar interface is simulated permanently. A snapshot is displayed in Fig. 1 using the turbulent vorticity $\zeta = (\nabla \times (\mathbf{u} - \mathbf{U}_L))_z$. This figure shows that there are three regions: weak turbulence ($x \lesssim 20$), chaotic interface ($20 \lesssim x \lesssim 34$), and laminar ($34 \lesssim x$) regions. Moreover, the chaotic interface contains dynamic inner structures. The snapshot shows a meandering bulk structure and strong wall shear layers. The weak turbulence consists of spatially modulated chaotic TS-waves. The chaotic interface is nothing but FCS, and we will reveal in the following that it generates the weak turbulence. We first investigate the energy balance of these regions, and then construct a phenomenology for its self-sustaining mechanism and functionality.

To focus on its streamwise inhomogeneity, we consider

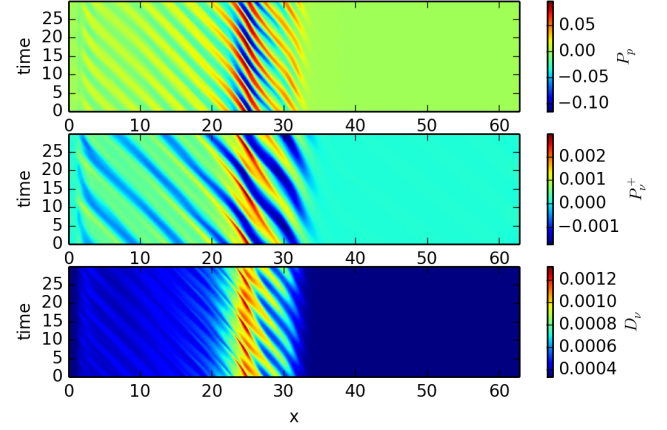


FIG. 2. x - t plot of P_p , P_v^+ , and D_v . The characteristic structure around $x \sim 25$ corresponds to the vortex ejection processes.

the y -averaged energy balance equation:

$$\frac{\partial E}{\partial t} + \partial_x (J_u + J_v) = P_p + P_v - D_v + F.$$

It should be noted that the energy is defined in the interface frame: $E(x, t) = \int_{-1}^1 dy \|\mathbf{u}\|^2 / 2$. Since the walls move, there is an energy injection due to the viscosity on the walls $P_v = P_v^+ + P_v^-$, where $P_v^\pm = \mp c_I \partial_y u_x|_{y=\pm 1} / Re$ in addition to the bulk viscous dissipation

$$D_v = \frac{1}{Re} \int_{-1}^1 dy \left(2 (\partial_x u_x)^2 + (\partial_x u_y)^2 + (\partial_y u_x)^2 \right).$$

The term $P_p(x, t) = - \int_{-1}^1 dy (\mathbf{u} \cdot \nabla) p$ represents the energy injection due to the pressure gradient, and takes both positive and negative values. $P_p > 0$ means the flow accelerated by the pressure gradient, and $P_p < 0$ does the flow against the pressure gradient. P_p balances almost with the gradient of the energy flux $\partial_x J_u$ and their spatial means are smaller than those of the viscous terms P_v and D_v . The flux due to the viscosity J_v is negligible, and thus neglected hereafter. F is the energy damping by the filter term. The three terms P_p , P_v^+ , and D_v are displayed in Fig. 2, which illustrates the traveling of each structures. Reflecting the chaotic nature of the interface, these values are not exactly periodic. $P_v^-(x, t)$ nearly equals to $-P_v^+(x, t + T_p/2)$, where $T_p \sim 15$ denotes an approximate period of the recurrent motion at each

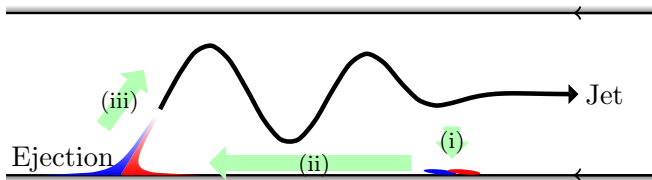


FIG. 3. A schematic view of the ejection-jet cycle.

point, and thus P_ν is recurrent with the half period $T_p/2$ like as P_p and D_ν .

To confirm that the chaotic interface maintains itself in terms of energy balance, the energy balance equation is averaged over the interface region $x \in [20, 34]$:

$$\frac{dE_I}{dt} + \Delta J_u = P_{p,I} + P_{\nu,I} - D_{\nu,I},$$

where the inferior \cdot_I denotes the average over the interface, and $\Delta J_u(t) = J_u(34, t) - J_u(20, t)$. The time average $\langle \cdot \rangle$ of these terms are calculated: $\langle dE_I/dt \rangle = -6.7 \times 10^{-6} \approx 0$, $\langle \Delta J_u \rangle = 3.0 \times 10^{-4}$, $\langle P_{p,I} \rangle = 4.3 \times 10^{-4}$, $\langle P_{\nu,I} \rangle = 1.2 \times 10^{-3}$, and $\langle D_{\nu,I} \rangle = 1.3 \times 10^{-3}$. It should be noted that there is the averaged energy leak $\langle \Delta J_u \rangle > 0$, which means that the chaotic interface is self-sustainable in terms of the time-averaged energy balance, and even an energy supplier to the weak turbulence. This energy leak reflects the functionality of the chaotic interface, i.e. the chaotic interface sustains the weak turbulence. The right after region of the chaotic interface has larger energy or stronger turbulent intensity than the downstream side of the weak turbulent region or asymptotic chaotic TS-wave. This convective relaxation process from this energy excess state to the asymptotic chaotic TS-wave state is similar to a temporal relaxation process of a minimal 2D channel flow, which is not shown in this paper. This similarity and the relationship to the phase reduction scenario is left to future works.

EJECTION-JET CYCLE

Here we give a concrete description of the self-sustaining mechanism of the chaotic interface. This sustaining process is constituted by the interaction among vortex ejections on the walls and the meandering jet in the bulk region. This collective dynamics is further split into three steps as summarized in Fig. 3. In the step (i), a pair of sheet-like vortices is excited by the instability of the laminar flow near the wall triggered by the meandering jet. The amplitude of the meandering decays, and the jet gets straight as going upstream. This suggests that a straight jet is convectively stable. Since the straight jet does not excite the vortex pair, it does not appear in $x > 35$.

The step (ii) is the convective growth of the vortex pair. The thin vortex pair generated in the step (i) grows up

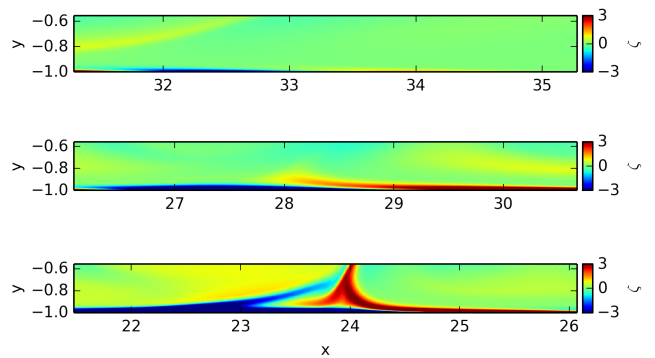


FIG. 4. The growth of the vortex pair. Three figures display different parts of the same snapshot. Regions $\zeta > 3$ ($\zeta < -3$) are colored by the same color of $\zeta = 3$ ($\zeta = -3$) to emphasize the bulk vorticity, though the vorticity field $\zeta(x, y)$ reaches its maximum $\zeta_{\max} \simeq 25$ around $x \simeq 24$.

into an intense vortex ejection. This process is displayed in Fig. 4, which picks up three continuing parts from a snapshot. Since the vortex pairs grow convectively, one snapshot of the entire channel gives three snapshots of the growing vortex pairs. In the energy viewpoint, it should be emphasized that this instability is not absolute but convective in both the interface frame and the laboratory frame. In the laboratory frame, the vortex pair goes upstream at a constant speed $c_v \simeq 0.5 > 0$, and, in the interface frame, it goes downstream at $c_v - c_I \simeq -0.35 < 0$.

The step (iii) is the vortex ejection process, which excites the jet and makes it meander. The ejection occurs on the downstream side of the chaotic interface, namely around $22 < x < 28$. Then the cycle is closed, and we call this cycle an “ejection-jet” cycle (EJC). A very strong shear accompanies this vortex ejection process. The wall unit l_τ is estimated at 2.1×10^{-3} , and the friction Reynolds number $Re_\tau = l_\tau^{-1}$ is about 460. This means that the width of the interface is 5000 times larger than l_τ . Therefore, we should regard this interface structure as a large-scale motion in the wall-turbulence context. After the intensive ejection process, the vortex structures are swept downstream, and this corresponds to the leak of the energy $\langle \Delta J_u \rangle$ from the interface to the weak turbulence region.

To complete the EJC model, let us consider how the invading speed c_I is determined. There are two dynamical processes, the convective growth of the vortex pair and the decay of the jet meandering. First, we suppose that both the traveling speed of each vortex pair c_v and the period T_v necessary to grow up are constant. From Fig. 2 we estimate them at $c_v \simeq 0.5 \pm 0.05$ and $T_v \simeq 20 \pm 2$ [18]. Their inaccuracies are due to the inaccurate definitions of them, and more accurate and quantitative arguments are left to future works. Then $d(c_I) := |c_v - c_I|T_v$ denotes the distance between the birth point of the vortex pair and its ejection point. Next, we introduce a characteristic length

λ of the decay of the meandering. Since this process is a nonlinear energy redistribution, we cannot define it from the spatial linear decay rate, but instead we measure the distance between the point where $\langle \max_y |u_y(x, y)| \rangle$ takes its maximum ($x = 25.4$) and the point where it becomes almost zero first ($x \simeq 38 \pm 2$). Since no vortex pair is excited when the jet does not meander, the EJC model requires these two length are equal:

$$|c_v - c_I|T_v = \lambda.$$

This condition connects two values characterizing the different dynamical processes, and thus we should regard this condition as a self-consistent equation for c_I . The above estimates are consistent with $c_I = 0.855$.

Let us review the EJC model by introducing filtered simulations. We make other three runs in which the filtered region Ω is set to damp one of the specific processes, namely, (a) weak turbulence, (b) vortex ejection, and (c) vortex pair excitation. Although we cannot split out each primary dynamics completely, these filtered simulations help us confirm the EJC model. These simulations use a snapshot of the previous simulation as an initial value, and animations visualized by the turbulent vorticity are included in the supplementary materials.

Case (a): we set $\Omega^{(a)} = [0, 22] \times [-1, 1]$ to damp the weak turbulent region, and to confirm the self-sustainability of the chaotic interface. In this setting we yield a permanent chaotic interface, whose invading speed and the spatial structure are hardly changed. We conclude that the following weak turbulence is additional as assumed in the EJC model. Furthermore, the selection process of c_I and the spatial structure is completely closed in the chaotic interface. In other words, the weak turbulence region plays no role in the selection process.

Case (b): we set $\Omega^{(b)} = [0, 30] \times [-1, 1]$ to confirm that the jet is maintained by the acceleration due to the vortex ejection. If the meandering jet is self-sustaining, this simulation could yield a permanent finite amplitude solution. However, the laminar flow has occupied whole region. In this sense, the meandering of the jet is only a component mechanism of this self-sustaining process, and is not self-sustaining.

Case (c): we set $\Omega^{(c)} = [30, 20\pi] \times [-1, 1]$ to obstruct the step (i). In this case the non-filtered region of the chaotic interface ($20 < x < 30$) keeps alive on the same position until $t \lesssim 20$, and then it travels downstream. This time lag corresponds to the growth time T_v of the vortex ejection, and thus this result also supports the EJC model. After a long transient, another chaotic interface is reconstructed around $15 \lesssim x \lesssim 27$, and their invading speed and spatial structure are same as the previous one. This result insists that the chaotic interface structure is robust while there is a laminar flow on its upstream. This robustness is an important issue for the pattern selection problem, but the current framework of

the dynamical systems approach lacks tools applicable for settling the issue.

CONCLUDING REMARKS

We have investigated the self-sustainability and functionality of CI in two-dimensional channel flow as an example of FCS, which yields the inhomogeneity between two asymptotic homogeneous states, upstream laminar flow and downstream chaotic TS-wave. We have introduced a phenomenology summarized in the EJC model, which consists of the vortex ejection and the meandering jet. The localized dynamics of CI is isolated by the filtered simulation, and deconstructed by the energy balance analysis. The EJC model well represents both the invading process on its front and its functionality in sustaining the weak turbulence on its tail. As a result, however, this functionality prevents us from obtaining an exact localized solution corresponding to the chaotic interface as done for various coherent structures because weak turbulence must attach to the interface. The damping filter works effectively in isolating the localized dynamics of CI.

The self-sustaining mechanism described by the EJC model is also an example for collective dynamics of multiscale structures. Different from Waleffe's self-sustaining process [19] which utilizes an absolute instability, the EJC model does a convective instability, which needs a sufficient space to grow up. The convective instability makes it possible for the structures of different scales to interact with each other, namely the meandering jet of large scale and the wall shear of small scale. This multiscale interaction mechanism may be applied for the large-scale motion in three-dimensional wall-turbulence [20], although the chaotic nature of CI is far weaker than that of three-dimensional wall-turbulence. Furthermore, it may also be a prototype for more general multiscale collective dynamics.

We have introduced the functional coherent structure (FCS), which extends the well-known coherent structure perspective. Previous studies have focused on the self-sustainability of coherent structures, but we do on its additional functionality. We expect that the idea to assign functionalities of turbulence to localized coherent structures may work well for other cases. Energy and momentum transfers in fully-developed wall-turbulence are possible applications since functional Waleffe's SSP, if it exists, may be embedded near the wall. For further development of the building block strategy, we will have to combine this functional building block scenario with the phase reduction scenario. In other words, we have to establish a framework involving phenomenological low-dimensional models of FCS and their interactions, and it is left to future works. This framework will be an essential tool for the dynamical systems approach to inho-

mogeneous turbulence and more general spatiotemporal chaotic systems.

This work is supported by the Grant-in-Aid for JSPS Fellows No. 26-1005 and the Grants for Excellent Graduate Schools “The Next Generation of Physics, Spun from Universality and Emergence” from the Ministry of Education, Culture, Sports, Science, and Technology (MEXT) of Japan, and also partially by JSPS KAKENHI Grant Number 22540386. A part of numerical calculations were carried out on SR16000 at YITP in Kyoto University.

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